

From Depth-Averaged Wave Models to Quasi-3D Transport: A Framework for Nearshore Microplastics

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ABSTRACT

Understanding microplastics transport in the nearshore zone requires an accurate representation of wave-induced circulation and vertical mixing processes. This study presents a methodological framework for reconstructing continuity-consistent quasi-three-dimensional velocity fields from depth-averaged outputs of the Nonlinear Shallow Water (NSW) and Boussinesq equations. A benchmark configuration is first employed to verify the numerical consistency and physical plausibility of the model. The framework is then extended to compare shallow water and Boussinesq formulations over a simplified planar bathymetry, enabling the influence of individual equation terms on tracer evolution to be isolated and physically interpreted. The proposed technique offers a valuable tool for extending the utility of efficient two-dimensional models toward quasi-three-dimensional hydrodynamic analysis.

Keywords: Vertical Velocity field reconstruction, Shallow water equations, Boussinesq equations, Nearshore hydrodynamics, Wave-induced circulation, Microplastics transport.

1. Introduction

Depth-averaged wave models such as the Nonlinear Shallow Water (NSW) and Boussinesq equations are widely used in nearshore hydrodynamics due to their computational efficiency, but their depth integration removes explicit information on vertical flow structure. This limitation can strongly affect particle and microplastics transport predictions, as similar depth-averaged circulation patterns may lead to markedly different particle trajectories and accumulation zones (Lesser et al., 2004; Soori et al., 2025). In the surf zone, wave transformation and breaking-induced circulation further emphasize the importance of vertical mixing and wave-induced kinematics for buoyant particle transport (Reniers et al., 2004; Kim and Kim, 2024). To address this limitation, this study develops a continuity-consistent framework to reconstruct three-dimensional velocity fields from depth-averaged NSW and Boussinesq outputs, verified using a benchmark configuration and applied to an idealized planar bathymetry to isolate the role of dispersive effects in nearshore transport processes. The focus is on physical interpretation rather than code benchmarking.

2. Methodology

To benchmark the proposed methodology, simulations are conducted over a controlled, idealized two-dimensional bathymetry $h(x, y)$ (Fig. 1) which allows isolating wave-induced circulation and vertical velocity. A vertically shifted coordinate system ($z^* = z + C; C = h(x_s, y_s) = \text{constant}$) is introduced to simplify the application of boundary conditions while preserving vertical derivatives.

2.1. Velocity Reconstruction for NSW Equations

The NSW model assumes a depth-uniform horizontal velocity, $u(x, y, z^*, t) = \bar{U}(x, y, t)$. The vertical velocity w is reconstructed by vertically integrating the local form of the continuity equation from the seabed upwards:

$$w(x, y, z^*, t) = w_b(x, y, t) - \int_{z_b^*(x, y)}^{z^*} (\partial u / \partial x + \partial v / \partial y) d\zeta \quad (1)$$

where $w_b(x, y, t) = -u_b(x, y, t) \frac{\partial h}{\partial x} - v_b(x, y, t) \frac{\partial h}{\partial y}$ is the kinematic bottom boundary condition for a fixed, impermeable bed.

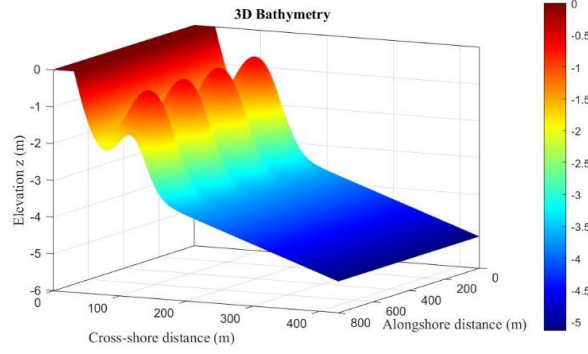


Fig. 1. Two-dimensional bathymetry $h(x,y)$ adopted for benchmark simulations (Kim and Kim, 2024).

2.2. Velocity Reconstruction for Boussinesq Equations

The Boussinesq model provides depth-averaged horizontal velocities and includes $O(\mu^2)$ dispersive terms that introduce weak vertical shear. The horizontal velocity profile is reconstructed as:

$$u(x, y, z^*, t) = \bar{u} - \mu^2 \left[h A_u \left(\frac{h}{2} + z^* - C \right) - B_u \left(\frac{h^2}{3} - (z^* - C)^2 \right) \right] + O(\mu^4) \quad (2)$$

$$A_u = \frac{\partial}{\partial x} \left[(h\bar{u})_x + (h\bar{v})_y \right], \quad B_u = \frac{\partial}{\partial x} (\bar{u}_x + \bar{v}_y)$$

The reconstruction of the transverse velocity component follows the same approach. Corresponding vertical velocity (w) is obtained by integrating the continuity equation using u and v profiles, analogous to Eq. (1).

3. Results and Discussion

A comparison between the reconstructed NSW and Boussinesq velocity fields highlights the role of dispersive effects. While the NSW reconstruction yields a strictly depth-uniform horizontal velocity and a linear vertical velocity profile, the Boussinesq reconstruction introduces weak vertical shear in the horizontal velocity and weak deviations from the strictly linear NSW profile, reflecting the influence of wave dispersion and bathymetric gradients. The reconstructed velocity fields provide the hydrodynamic basis for the subsequent particle-tracking simulations. In particular, the contrast between the depth-uniform NSW velocities and the weakly sheared, continuity-consistent Boussinesq velocities directly governs the ability of the model to represent vertical particle displacement, suspension, and cross-shore redistribution of microplastics particles. After benchmarking the NSW and Boussinesq reconstructions over a on real-field scale sandbar beach, preliminary results indicate physically consistent behavior. An idealized planar bathymetry is then used to further isolate the influence of individual equation terms on tracer evolution.

4. Conclusion

This study presents a diagnostic framework for reconstructing three-dimensional velocity fields from two-dimensional outputs of the NSW and Boussinesq models by enforcing the incompressibility condition in a shifted vertical coordinate system. In the NSW model, the reconstruction confirms a depth-uniform horizontal velocity and a linearly varying vertical velocity, fully consistent with the model assumptions. For the Boussinesq model, weak vertical shear appears in the horizontal velocity due to dispersive effects, while the vertical velocity remains governed by the continuity equation. By providing physically consistent vertical velocity structure, the proposed reconstruction improves the hydrodynamic representation of microplastics transport, including vertical displacement, suspension, and cross-shore redistribution in the nearshore zone including offshore-directed undertow within the wave-breaking region.

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